AAPT | UNITED STATES PHYSICS TEAM

AIP | 1999

Solutions to Problems

Any correct, equivalent solution should be awarded proportionate points. You may further break down the listed points into one point increments. Students should not be penalized in a subsequent part for using the wrong answer to a previous part. (No double jeopardy.)

Points

1. $y_{max} = 10$ m $v_o = 25$ m/s $a = 2$ m/s^2

a. The train's acceleration does not affect the vertical motion.

At the highest point, $v_y = 0$ 2

$$v_y^2 = v_{oy}^2 - 2gy$$ 2

$$v_{oy} = \sqrt{2gy_{max}} = \sqrt{2(9.8m/s^2)(10m)} = 14m/s$$ 2

Since
$$v_{oy} = v_o \sin\theta$$ 2

$$\theta = \sin^{-1}\left(\frac{v_{oy}}{v_o}\right) = \sin^{-1}\left(\frac{14m/s}{25m/s}\right) = 34.1^o$$ 2

b. The time to reach y_{max} is obtained from $v_y = v_{oy} - gt$ at the highest point

$$t = \frac{v_{oy}}{g} = \frac{14m/s}{9.8m/s^2} = 1.43s$$ 5

The time to land is twice that, or $t = 2.86$ s. 2

During that time, the ball moves a distance $x_b = v_{oc}t - v_o\cos\theta t$ 2

where v_{oc} is the velocity of the train with respect to the Earth when the ball is launched.

And the front end of the train moves $x_p = v_{oc}t + \frac{1}{2}at^2$ 2

The difference of those displacements is the ball's displacement with respect to the train.

$$\Delta x = x_b - x_p = -v_o\cos\theta t - \frac{1}{2}at^2$$ 2

$$\Delta x = -25m/s(\cos 34.1^o)2.86s - \frac{1}{2}(2m/s^2)(2.86s)^2 = -67.3m$$ 2

(Either sign okay. 67.4 m also okay. Part b may also be solved in the train's accelerating frame or in an inertial frame with the same velocity with respect to the Earth that the train had when the ball was launched. Any correct method may be used.)

2. a. Apply energy conservation to equate total energy at point A and point B. 2

$$4MgR = 2MgR + \frac{1}{2}Mv^2$$ 3

$$Mv^2 = 4MgR$$ 1

Applying Newton's 2nd law to the forces at point B 2

$$N + Mg = M\frac{v^2}{R}$$ 4

$$N + Mg = \frac{(4MgR)}{R}$$ 1

$$\text{or } N = 3Mg \qquad \text{downward}$$ 2

b. Since it is rolling at point B, it also has rotational kinetic energy there

$$4MgR = 2MgR + \tfrac{1}{2}Mv^2 + \tfrac{2}{5}MR^2\omega^2$$ 3

With $\omega = v/R$ 3

$$2MgR = (\tfrac{1}{2} + \tfrac{2}{5})Mv^2 = \tfrac{7}{10}Mv^2$$ 1

$$N + Mg = \frac{(\tfrac{20}{7}MgR)}{R}$$ 1

Thus $N = \tfrac{13}{7}Mg$ downward 2

3. Let h_i = 0.20 m the initial height of m_1.
   - h_f = the final height of both masses.
   - v_{1i} = the speed of m_1 just before the collision.
   - v_{1f} = the speed of m_1 just after the collision.
   - v_{2f} = the speed of m_2 just after the collision.

a. Before the collision, for m_1 $\quad m_1gh_i = \tfrac{1}{2}m_1v_{1i}^2$ 3

After the collision, for m_1 $\quad m_1gh_f = \tfrac{1}{2}m_1v_{1f}^2$

After the collision, for m_2 $\quad m_2gh_f = \tfrac{1}{2}m_2v_{2f}^2$

Since both masses rise to the same height after the collision, they must have equal speeds after the collision $\quad v_{1f} = v_{2f} = v_f$ 3

Since the collision is elastic, the total kinetic energy is unchanged 2

$$\tfrac{1}{2}m_1v_{1i}^2 = \tfrac{1}{2}m_1v_f^2 + \tfrac{1}{2}m_2v_f^2$$ 2

or

$$m_1gh_i = m_1gh_f + m_2gh_f$$

$$m_1h_i = (m_1 + m_2)h_f \qquad \text{Eq. 1}$$ 2

Using momentum conservation 2

$$m_1v_{1i} = m_2v_f - m_1v_f$$ 2

or

$$m_1\sqrt{2gh_i} = (m_2 - m_1)\sqrt{2gh_f}$$

$$m_1\sqrt{h_i} = (m_2 - m_1)\sqrt{h_f}$$

Squaring both sides $\quad m_1^2h_i = (m_2^2 - 2m_1m_2 + m_1^2)h_f$

Substituting inEq. 1 $\quad m_1(m_1 + m_2)h_f = (m_2^2 - 2m_1m_2 + m_1^2)h_f$

Solving for the relation between the masses $\quad 0 = m_2^2 - 3m_2m_1$

Or assuming $m_2 \neq 0$ $\quad m_2 = 3m_1$ 2

Solving Eq. 1 for h_f $\quad h_f = \frac{m_1}{m_1 + m_2} h_i = \frac{m_1}{m_1 + 3m_1} h_i = \frac{1}{4} 0.20m = 0.050m$ 2

b. After the second collision m_2 stops 3

and m_1 reaches maximum height h_i 2

4. Imagine the door to be a compound pendululm in a gravitational field whose "g" equals the car's acceleration. We are interested in finding 1/4 of the period of the oscillation, the time it takes the door to go from a maximum small amplitude to its equilibrium position. The torque and rotational inertia are:

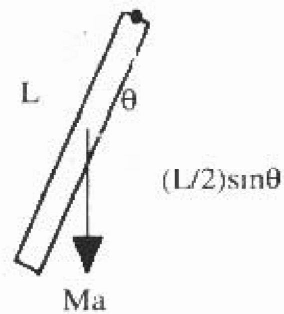

Diagram similar to accompanying 5

$$\tau = -Ma\left(L/2\right)\sin\theta$$ 3

$$I = (1/3)ML^2$$ 4

$$(1/3)ML^2\alpha = -Ma\left(L/2\right)\sin\theta$$ 3

For small angles we have $\quad \alpha \cong -(3/2)\left(a/L\right)\theta$ 1

$$\omega^2 = \frac{3a}{2L}$$ 3

$$t = \frac{T}{4} = \frac{1}{4}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{2}\sqrt{\frac{2L}{3a}}$$ 2

The distance the car travels is $\quad D = \frac{1}{2}at^2 = \frac{1}{2}a\frac{\pi^2}{4}\left(\frac{2L}{3a}\right) = \frac{\pi^2}{12}L$ 4